REAL TIME WIDEBAND CEPSTRUM ANALYSIS BASED ON SAW DEVICES

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ABSTRACT

Cepstrum analysis using Surface Acoustic Wave Fourier (Chirp) Transform processors has application in waveform detection and classification. Analyser capabilities are demonstrated theoretically and practically measuring the pulse width, duration, period and frequency of unknown input waveforms.

INTRODUCTION

Cepstrum analysis[1] has application in waveform identification[2] for target classification, signal extraction in multipath environments[3], and in speech processing[4]. To date, signal processing based on cepstrum techniques has been largely confined to computer based systems. This paper illustrates how recent developments in Surface Acoustic Wave (SAW) device technology can be used to implement *real time wideband* power cepstrum analysis with projected application in sophisticated signal processing for radar, sonar and communications systems. The design and performance of an original, prototype real time power cepstrum analyser are described. The system uses two SAW Fourier Transform processors based on the Chirp Transform algorithm. Practical results are included which demonstrate the operational capabilities of the SAW based cepstrum analyser when processing either IF or baseband waveforms.

PRINCIPLES OF CEPSTRUM ANALYSIS

Cepstrum analysis is, by definition[1], achieved by a serial arrangement of two Fourier Transform processors. The first processor transforms from the time domain to the frequency domain, yielding the spectrum of the input waveform. After signal amplification in a true logarithmic amplifier, the second processor transforms from the frequency domain to a pseudo-time (quefrency) domain, to yield the cepstrum of the input waveform. For certain waveforms, particularly time periodic and superimposed time coincident waveforms, spectrum analysis can yield an ambiguous display. The deconvolution effect achieved by logarithmic processing permits detailed examination of the features of the spectrum.

A simplified description of the operation of the cepstrum analyser is given in Figure 1 for a pulse of duration T with an echo at epoch (delay) τ' relative to the main pulse, Figure 1(a). The spectrum, Figure 1(b) shows a basic sinc function response, characterised by the signal duration T, with a superimposed sinc function characterised by the echo epoch τ'. The effect of the logarithmic amplifier is to decompose the spectral effects of pulse and echo, forcing a more nearly cosinusoidal amplitude variation with frequency, Figure 1(c). The log spectrum is therefore itself periodic in the frequency domain.

The power cepstrum[2], $C(\tau)$, has been defined as the power spectrum of the logarithmic power spectrum of an input signal $f(t)$:-

$$C(\tau) = |FT\{\log|FT\{f(t)\}|^2\}|^2 \qquad (1)$$

where FT denotes Fourier Transformation. The second Fourier Transform effectively analyses the

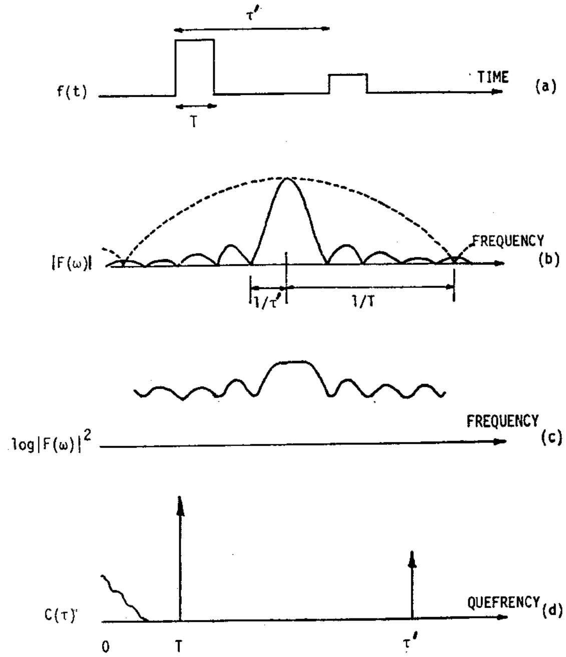


FIGURE 1. PRINCIPLES OF CEPSTRUM ANALYSIS.

frequency periodic components of the log power spectrum to yield the cepstrum. The term quefrency[1] is used in the cepstrum and corresponds to frequency in the spectrum. Quefrency, which is effectively a measure of time period, has units of seconds. Figure 1(d) shows the cepstrum of the waveform under consideration with responses at three different quefrencies. The zero quefrency response results from the constant D.C. offset in the log power spectrum, while the other two responses correspond to the quefrencies produced by the basic input pulse and echo.

SAW CEPSTRUM ANALYSER DESIGN AND PERFORMANCE

SAW technology now permits through the ready availability of compact chirp filters[5], the realisation of a Fourier Analyser[6] using three identical filters plus associated amplifiers and timing electronics. The effective parallel processing during convolution in the SAW chirp filter permits real time analysis over signal bandwidths exceeding 10 MHz. Power cepstrum analysis can be performed by coupling two SAW Chirp Transform spectrum analysers through a true logarithmic amplifier, Figure 2. The four SAW chirp filters employed in our demonstration cepstrum analyser system were fabricated on ST,X quartz with impulse response duration of 5 μs comprising a

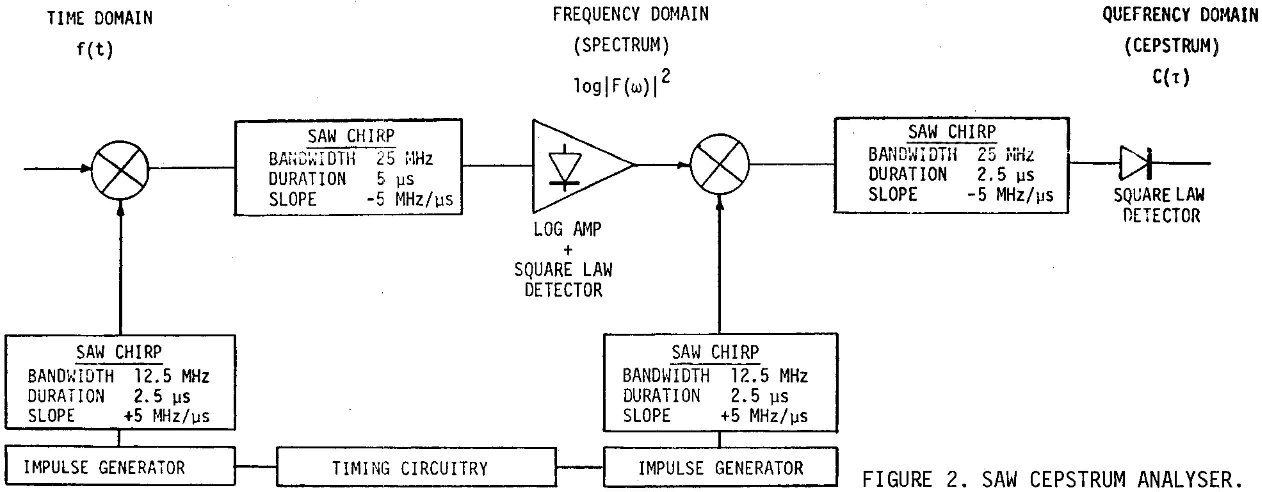


FIGURE 2. SAW CEPSTRUM ANALYSER.

linear frequency sweep over 25 MHz bandwidth centred at 60 MHz with a corresponding dispersive slope, μ, of 5 MHz/μs. The logarithmic amplifier used was a four stage Plessey SL530C amplifier. The first Chirp Transform processor[7] (spectrum analyser) transforms from the time domain to the frequency domain and is capable of analysing a bandwidth of 12.5 MHz in 2.5 μs with a CW frequency resolution of 400 kHz. The second Chirp Transform processor permits cepstrum analysis of quefrencies up to 2.5 μs with a resolution of 80 ns.

1. DETERMINATION OF PULSE DURATION

The power spectrum of a pulse of duration T which is obtained as a time function at the output of the SAW processor is given by

$$\left|\Phi(\omega)\right|^2 = \left|\Phi(\mu t)\right|^2 = \left|\frac{\sin(\mu tT)}{(\mu tT)}\right|^2 \qquad (2)$$

where μ corresponds to the dispersive slope of the SAW chirp filters, employed in the Chirp Transform processor. Thus the spectrum of the input signal is obtained at the output of the spectrum analyser through the relationship $\omega = \mu t$. Logarithmic amplification yields

$$\log\left|\Phi(\mu t)\right|^2 = 2\log[\sin(\mu tT)] - 2\log[\mu tT] \quad (3)$$

which is periodic in the frequency domain with period $\omega = \mu t = 1/T$. Since the log spectrum approximates to a sinusoid, further analysis yields a single cepstrum response whose quefrency is determined by T, the basic pulse duration.

Figure 3 shows the operation of our demonstration SAW cepstrum processor with such a pulse waveform. The spectrum of a T = 1 μs pulse measured at the output of the first SAW Chirp Transform processor demonstrates the characteristic sinc function envelope with nulls spaced at 200 ns which corresponds to a frequency interval of 1 MHz (1/T) for this SAW Chirp Transform processor which employs filters with dispersive slope μ = 5 MHz/μs. The waveform envelope after logarithmic amplification shows the spectrum envelope to be more nearly cosinusoidal, with period 200 ns. The second Chirp Transform processor therefore effectively sees a 5 MHz CW signal and performs a spectrum analysis of this waveform. The output of the second Chirp Transform processor corresponds to the cepstrum of the input signal to the first Chirp Transform processor with responses at $\tau = \pm 1$ μs and can be used to directly measure pulse width.

Note the presence of the zero quefrency cepstral response. The variations in amplitude between the positive and negative cepstrum peaks is attributable directly to a non-uniform amplitude response in the second SAW convolutional chirp filter.

The described SAW filter parameters permit our cepstrum analyser to determine pulse durations up to the 2.5 μs duration of the premultiplier chirp. In practice, the zero quefrency term present in the cepstrum limits the minimum resolvable pulse duration to typically 150 ns. Our SAW processor, with a limited time bandwidth product of 32, is therefore capable of determining pulse duration over a range in excess of 10 to 1.

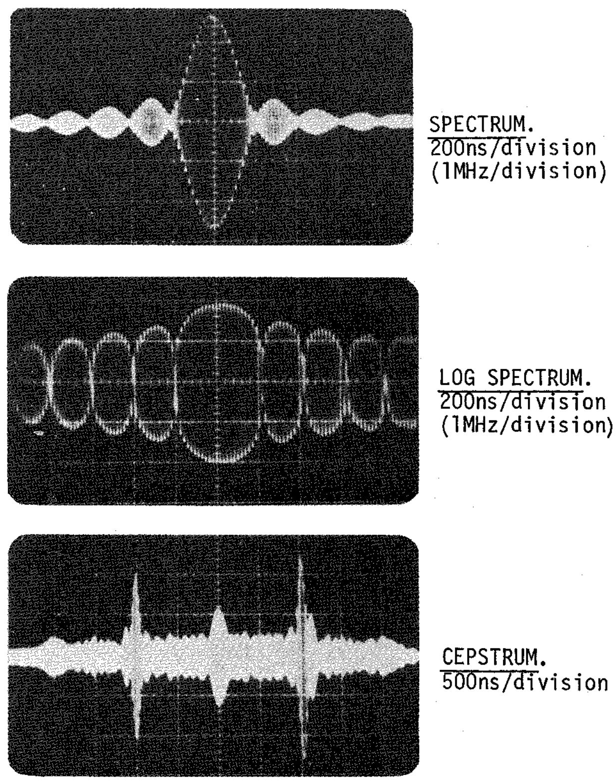


FIGURE 3. CEPSTRUM DETERMINATION OF PULSE WIDTH.

In comparison to alternative techniques for the determination of pulse length (such as differentiation), the SAW cepstrum processor provides signal to noise ratio (SNR) enhancement. The first Fourier Transform processor provides a maximum SNR in the spectrum display at the peak of the sinc response. Here the pulse compression provides a processing gain equal to 10 $\log_{10}\mu T^2$ where T is the input pulse duration. As the input duration T is reduced the SNR improvement reduces. However, the lobes of the sinc response widen, maintaining good average SNR over the output waveform. The effect of the logarithmic amplifier is to force the SNR to unity since low level signals are preferentially amplified. Typically, an SNR of -20 dB at the log amp input is reduced to -3 dB at the output. Thus in the cepstrum analyser, the second Chirp Transform processor operates with an input SNR close to unity such that the overall SNR improvement is approximately 10 $\log_{10}\mu T_c^2$. Here, T_c is the premultiplier duration. Note that increasing the time bandwidth product of the second Chirp Transform processor would tend to increase the processing gain of the cepstrum analyser and also permit investigation of the waveforms surrounding the nulls of the sinc response which are determined by the rise time and fall time of the input pulse.

2. DECOMPOSITION OF PULSE WITH DISTORTING ECHOES

The power spectrum of a signal s(t) distorted by an echo of amplitude α and relative delay τ' can be expressed as [2]

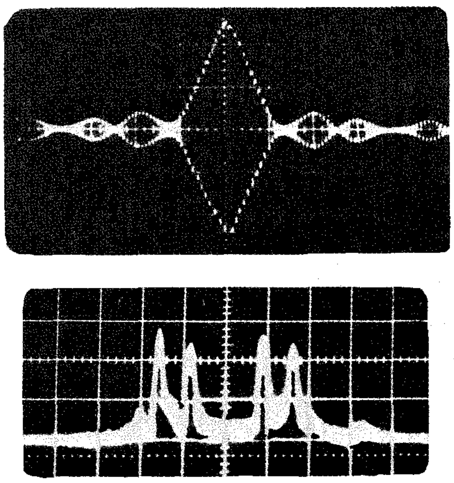
$$|X(\omega)|^2 = \Phi_s(1 + 2\alpha\cos\omega\tau' + \alpha^2) \qquad (4)$$

where $\Phi = |s(\omega)|^2$ (5)

which is seen to be a product of two terms, i.e. a frequency domain multiplication, or time domain convolution. These terms can be decomposed or deconvolved by taking the logarithm

$$\log|X(\omega)|^2 = \log\Phi + \log(1 + 2\alpha\cos\omega\tau' + \alpha^2) \qquad (6)$$

which indicates that the effect of the echo is to produce a cosinusoidal quefrency response of periodicity $1/\tau'$ corresponding to the echo epoch.



SPECTRUM. 200ns/division (1MHz/division)

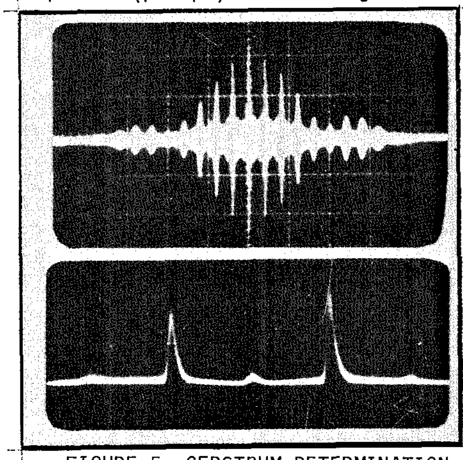
CEPSTRUM. 500ns/division

FIGURE 4. CEPSTRUM DECOMPOSITION OF PULSES.

Figure 4 shows the cepstrum of a pulse signal of duration T = 800 ns, distorted by an echo arriving at epoch τ' = 400 ns. The cepstrum shows one response corresponding to the basic pulse duration at 800 ns, with a further response at the echo epoch. Information regarding basic pulse length and echo epoch are not directly obvious in the signal spectrum. The 400 ns echo epoch condition corresponds to a self distortion of the waveform since the pulse and echo are partially time coincident. As the echo epoch varies, the position of the echo cepstral peak varies whilst the cepstral peak due to the basic pulse remains fixed.

3. MEASUREMENT OF PULSE REPETITION PERIOD

The spectrum and cepstrum of a 100 ns pulse train with repetition frequency (p.r.f.) of 1 MHz are shown in Figure 5. The spectrum shows the expected sinc function envelope with nulls at ± 10 MHz (± 2 μs x 5 MHz/μs), corresponding to the pulse width of 100 ns. The lines within this sinc envelope have separations determined by the input p.r.f. The power cepstrum of the 1 MHz waveform exhibits peaks at quefrency τ = ± 1 μs, corresponding directly to the input repetition period. Our processor permits measurement of pulse repetition period (p.r.p.) over a range 250 ns - 2.5 μs.



SPECTRUM. 1μs/division (5MHz/division)

CEPSTRUM. 500ns/division

FIGURE 5. CEPSTRUM DETERMINATION OF PULSE RATE.

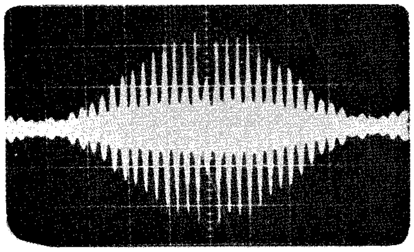
4. DETERMINATION OF BINARY CODE LENGTH AND BIT RATE

The power cepstrum can also be used to determine the code length of a binary code or the bit rate given *a priori* knowledge of one of these parameters. In principle it is possible to determine both parameters simultaneously. However, in practice, the limited processor time bandwidth product (32) and the fact that the input code was asynchronous relative to the processor, precluded this simultaneous measurement.

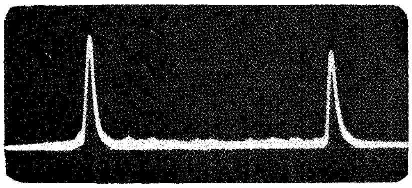
The spectrum of a 15 bit pseudo noise code at 10 MHz bit rate is shown in Figure 6. The first nulls of the sinc envelope are spaced at ± 10 MHz (± 2 μs x 5 MHz/μs) and the number of spectral lines to the first null is defined by the code length (15 bits). The power cepstrum, of this code is shown to consist of peaks at τ = 1.5 μs. This corresponds to the quefrency of a pseudo noise code of length n bits given by

$$\tau = (2^n - 1)/R \qquad (7)$$

where R is the bit rate.



SPECTRUM.
500ns/division
(2.5MHz/division)



CEPSTRUM.
500ns/division

FIGURE 6. CEPSTRUM ANALYSIS OF BINARY CODES.

5. DETERMINATION OF CHIRP SLOPE

A potential application of the SAW Cepstrum Analyser lies in the determination of the dispersive slope of an unknown radar chirp signal. A computer simulation has been undertaken to determine the capability of our SAW cepstrum analyser in this application. The power spectrum of any chirp signal of finite duration displays characteristic Fresnel ripples[8]. For a fixed duration chirp, an increase in dispersive slope, μ (and hence B) will increase the variation with frequency of the Fresnel ripples. This effect results in an increase in the quefrency of the unknown chirp spectrum. Hence the cepstrum of an unknown chirp waveform is predicted to exhibit a linear variation of quefrency with input chirp slope.

Computer simulated analysis of a range of input chirp dispersive slopes from 6 to 12 MHz/μs for the SAW processor described previously with μ = 5 MHz/μs has shown that the position of the largest peak in the cepstrum response is linearly dependent on the dispersive slope of the unknown chirp, Figure 7. We consider that detection, identification and classification of an unknown chirp waveform represents a potentially important application area for the SAW Cepstrum processor.

CONCLUSION

This paper has described the principles and demonstrated the performance of a wideband (MHz) real time Surface Acoustic Wave Cepstrum Analyser when processing waveforms typical of those encountered in radar and communication systems. The prototype analyser computes the cepstrum with two serial Fourier Transforms based on the Chirp Transform algorithm and is implemented with SAW chirp filters. The design parameters, B = 25 MHz T = 5 μs (TB = 125) of these devices permits the computation of a modest 32 point transform on 12.5MHz signal bandwidth. However, it demonstrates clearly the potential of SAW devices to implement wideband real time cepstrum analysers.

Our prototype analyser was shown to be effective in determining pulse duration and p.r.p. over a 10:1 range from 250 ns to 2.5 μs, this being limited by the TB product of the SAW chirp filter.

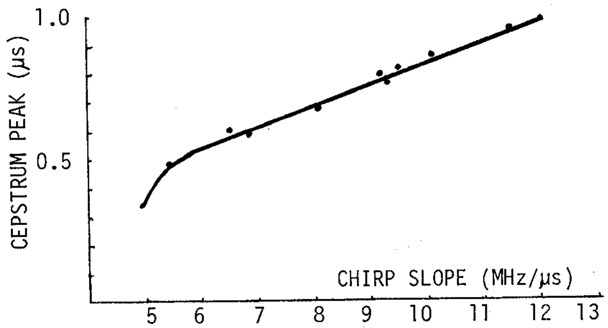


FIGURE 7. POSITION OF CEPSTRUM PEAK vs CHIRP SLOPE.

With currently available SAW chirp filters of impulse response durations ranging from 2 μs to 50 μs and TB products up to 10,000, SAW cepstrum analysers have the projected capability of measuring pulse durations and p.r.p. in the range 50 ns to 50 μs. The analyser performance with binary codes can be extrapolated to permit measurement of bit rates up to 200 MHz and code lengths up to 511. Computer simulation of the system performance in the determination of unknown chirp slope has demonstrated a capability of measuring chirp slope up to $\pm 2\ \mu$, where μ is the processor dispersive slope. In principle, cepstrum processors implemented with SAW devices can be designed for the determination of chirp slopes in the range 40 kHz/μs to 40 MHz/μs.

Real time, wideband SAW cepstrum analysers, which are an extension of existing compressive receiver designs, are seen as providing an additional technique for waveform classification and signal processing in ECM and ELINT applications to determine signal duration period and dispersive slope. Further the signal deconvolution properties of the cepstrum are of potential application in multipath environments such as low elevation search radar systems. In principle, target classification and identification might now be possible through the cepstrum analysis of target spectral characteristics such as engine sidebands.

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REFERENCES

1. Bogert, B P, Healy, M J and Tukey, J W, Proc Symp Time Series Analysis, Ch 15, pp 209-243, M Rosenblatt, Ed, New York, Wiley 1963.
2. Kemerait, R C and Childers, D G, IEEE Trans Information Theory, Vol IT-18, No 6, pp 745-759, November 1972.
3. Fjell, P O, J Acoust Soc Am, Vol 59, No 1, pp 209-211, January 1976.
4. Noll, A M, J Acoust Soc Am, Vol 41, No 2, pp 293-309 1967.
5. Gerard, H M et al, IEEE Trans MTT-21, No 4, pp 176-186, April 1973.
6. Otto, O W, Proc 1976 Ult Symp, IEEE Cat No CH1 120 5SU, pp 365-370.
7. Jack, M A et al, Proc 1976 Ult Symp, IEEE Cat No CH1 120 5SU, pp 376-381.
8. Cook, C E and Bernfeld, M, "Radar Signals", Academic Press, New York, 1967.